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13. ABSTRACT (Maximum 200 words)  There were two principal accomplishments during the grant period. The first gives a new more effective reordering role for processing jobs (that may not complete) on a multiprocessor system. This is shown to improve on the previously known MF (move to front) and MB (move to back) ordering rules. The other accomplishment was a new variance reduction method in simulation, using "random hazards" in a Markov process.				
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During this period 2 papers supported by the grant were published. In [1] the problem of determining efficient processor reordering rules were considered; and in [2] an important new tool for reducing the variance of estimators obtained by simulation was introduced. We now briefly describe these papers.

Suppose that  $n$  processors are arranged in an ordered list. When a job arrives, the first processor in line attempts to complete this job; if it is not successful in this attempt then the next one in line attempts to complete it, and so on. Each time processor  $j$  attempts to complete a job it is, independently of all that has occurred, successful with (an unknown) probability  $p_j$ ,  $j=1, \dots, n$ . After a job has been completed, or all  $n$  attempts have failed, then based on the result, we are allowed to reorder the list before the next job arrives; with the objective being to minimize the average number of attempts per job.

Two reordering rules for this problem had previously been considered in the literature for this problem. Namely, the "move to the front" and "move to the back" which we shall call MF and MB. The MF reordering rule moves a processor that has been successful to the front of the line, whereas the MB reordering rule moves each unsuccessful processor to the back of the line. It had been shown that MF dominates MB in that the long-run number of attempts per job is stochastically smaller under MF than under MB for any set of probabilities  $p_j$ ,  $j=1, \dots, n$ .

In [1] we introduced a new reordering rule, called the one-closer rule which moves the successful processor one closer to the front of the line, leaving the relative ordering of the other processors unchanged. We showed that the resulting Markov chain under this rule is time reversible and, using this, we were able to determine its stationary probabilities. We then considered the special case where all processors, but one, have the same success probability - that is, we supposed that  $p_j = p$ , for  $j \geq 2$ . In this case we showed that the one-closer rule results in a smaller average number of attempts per job than does the MF rule. This result was established by showing that if  $p_1 \geq p$  ( $p_1 \leq p$ ) then the asymptotic position of processor 1 is likelihood ordered ratio smaller (larger) under the one-closer rule than it is under the MF rule.

We also considered k-in-a-row rules in which a reorder is only made when the same result occurs k times in a row. It was then shown, in the special case of 2 processors, that the asymptotic efficiency of this rule increases to that of the optimal ordering as k increases to infinity.

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The paper [2] was concerned with obtaining improved simulation estimators by utilizing "random hazards". Consider a Markov process  $\{X_n, n \geq 0\}$  and for a pair of disjoint set of states A and B, let

$$N = \text{Min}\{n: X_n \in A \cup B\};$$

where we suppose that  $P\{N < \infty\} = 1$ . Let  $p = P\{X_N \in A\}$ . Let,

$$\begin{aligned} h_n &= P\{N = n | X_0, \dots, X_{n-1}\} \\ &= P_{X_{n-1}}(A), \quad \text{for } n \leq N \end{aligned}$$

where  $P_x(A)$  is the probability that the next transition is into A given that the present state is x. The quantities  $h_n$  are called random hazards, and the quantity H defined by

$$H = \sum_{n=1}^N h_n$$

is called the total hazard. It is easy to establish that

$$E[H] = P\{X_N \in A\} = p$$

It was shown in [2] that in cases where it is known that  $p=1$  and one is interested in estimating  $E[N]$  then H can be utilized as a powerful variance reduction control variate. Examples of its prowess as a control variate were presented in such areas as

- (a) computing the average run length of a moving average quality control chart;
- (b) analyzing Poisson shock models;
- (c) computing the expected failure time of a multicomponent system with repairs, where the repair rate of a component depends on the set of failed components and the order of their failure.

- (d) estimating the mean cycle time of a queueing system

In instances where one is interested in estimating  $p$  the total hazard can be utilized as an estimator. Examples were presented in [2] illustrating the strength of this estimator in

- (a) estimating the reliability function

and

- (b) estimating cycle probabilities in cumulative sum quality control charts.

#### REFERENCES

1. Ross, S., "Processor Reordering Rules", Prob. in Eng. and Inf. Sci., Vol. 4 No. 2, 181-186, 1990
2. Ross, S., "Variance Reduction in Simulation via Random Hazards", Prob. in Eng. and Inf. Sci., Vol. 4, No. 3, 299-309